Chapter 12 : The Triangle and Its Properties

ANSWER KEYS

EXERCISE 12.1

1. (*i*) Sum of the given angles = $76° + 64° + 40°$
$= 180°$

$\because$ The sum of the angles of a triangle is 180°.

Yes, the triplet (76°, 64°, 40°) can be the angles of a triangle.

(*ii*) Sum of the given angles = $30° + 70° + 90°$
$= 190° \neq 180°$

Hence, the triplet (30°, 70°, 90°) cannot be the angles of a triangle.

(*iii*) Sum of the given angles = $56° + 62° + 62°$
$= 180°$

Hence, the triplet (56°, 62°, 62°) can be the angles of a triangle.

(*iv*) Sum of the given angles = $20° + 148° + 30°$
$= 198° \neq 180°$

Hence, the triplet (20°, 148°, 30°) cannot be the angles of a triangle.

2. Let $\angle A$, $\angle B$ and $\angle C$ be the three angles of a triangle.

Let $\angle A = 60°$, $\angle B = 75°$, $\angle C = ?$

$\because$ Sum of the angles of a triangle is 180°.

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 60° + 75° + \angle C = 180°$

$\Rightarrow \quad 135° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 135°$

$\Rightarrow \quad \angle C = 45°$

Hence, the third angle is 45°.

3. Let the third angle be x.

Then, other two equal angles are $2x$ each.

$\therefore \quad 2x + 2x + x = 180°$

($\because$ Sum of the angles of a triangle is 180°)

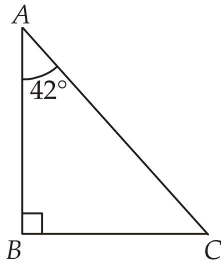
$\Rightarrow \quad 5x = 180°$

$\Rightarrow \quad x = \dfrac{180°}{5} = 36°$

Hence, the angles of the triangle are 72°, 72° and 36°.

4. Let ABC be a right angled triangle, right angle at B.

$\therefore \quad \angle B = 90°$



Let $\angle A = 42°$, $\angle C = ?$

$\angle A + \angle B + \angle C = 180°$ (Angle sum property)

$\Rightarrow \quad 42° + 90° + \angle C = 180°$

$\Rightarrow \quad 132° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 132°$

$\Rightarrow \quad \angle C = 48°$

Hence, the other acute angle is 48°.

5. Let all the three angles of a triangle be x.

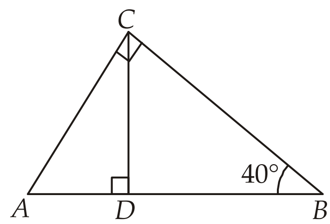
$\Rightarrow \quad x + x + x = 180°$ (Angle sum property)

$\Rightarrow \quad 3x = 180°$

$\Rightarrow \quad x = \dfrac{180°}{3} = 60°$

Hence, the measure of each angle is 60°.

6. (*i*)



In right angled triangle ABC, $\angle C = 90°$

$\because \quad \angle BAC + \angle ACB + \angle CBA = 180°$

$\Rightarrow \quad \angle BAC + 90° + 40° = 180°$

$\Rightarrow \quad \angle BAC = 180° - 130° = 50°$

Hence, $\boxed{\angle BAC = 50°}$

(*ii*) In ΔADC,

$\therefore \quad \angle CAD = \angle BAC$

$\angle CAD = 50°$

$\angle D = 90°$

$$\angle CAD + \angle ADC + \angle ACD = 180°$$
$$50° + 90° + \angle ACD = 180°$$
$$\Rightarrow \quad \angle ACD = 180° - 140° = 40°$$
Hence, $\boxed{\angle ACD = 40°}$

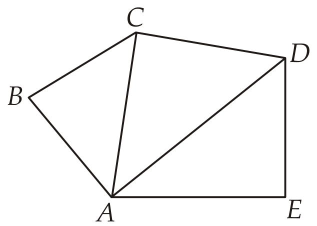
(*iii*) In ΔBDC,
$$\angle CBD + \angle BDC + \angle DCB = 180°$$
$$40° + 90° + \angle DCB = 180°$$
$$\Rightarrow \quad \angle DCB = 180° - 130° = 50°$$
Hence, $\boxed{\angle DCB = 50°}$

7. Let three angles of a right angled triangle be $\angle A$, $\angle B$ and $\angle C$.

Let $\angle A = \angle B = x$ and $\angle C = 90°$.
$$\angle A + \angle B + \angle C = 180° \text{ (Angle sum property)}$$
$$\Rightarrow \quad x + x + 90° = 180°$$
$$\Rightarrow \quad 2x = 180° - 90°$$
$$\Rightarrow \quad x = \frac{90°}{2} = 45°$$
$\Rightarrow \quad \angle A = \angle B = 45°$.

Hence, the two equal acute angles are 45°.

8.



In ΔABC,
$$\angle CAB + \angle ABC + \angle BCA = 180° \quad (\because \text{ Angle sum property}) \quad \ldots(i)$$
In ΔACD,
$$\angle DAC + \angle ACD + \angle CDA = 180° \quad \ldots(ii)$$
In ΔADE,
$$\angle EAD + \angle ADE + \angle DEA = 180° \quad \ldots(iii)$$
Adding (*i*), (*ii*) and (*iii*), we get
$$\angle CAB + \angle ABC + \angle BCA + \angle DAC + \angle ACD + \angle CDA + \angle EAD + \angle ADE + \angle DEA = 540°$$
$$(\angle CAB + \angle DAC + \angle EAD) + \angle ABC + (\angle BCA + \angle ACD) + (\angle CDA + \angle ADE) + \angle DEA = 540°$$
$$\angle EAB + \angle ABC + \angle BCD + \angle CDE + \angle DEA = 540°$$
$(\because \quad \angle CAB + \angle DAC + \angle EAD = \angle EAB,$
$\angle BCA + \angle ACD = \angle BCD,$
$\angle CDA + \angle ADE = \angle CDE)$

9. Let the three angles of a triangle be $5x$, $4x$ and $3x$. Therefore,
$$5x + 4x + 3x = 180° \quad (\because \text{ Angle sum property})$$
$$\Rightarrow \quad 12x = 180°$$
$$\Rightarrow \quad x = \frac{180°}{12} = 15°$$
Hence, the angles of a triangle are:

$5x = 5 \times 15° = 75°$

$4x = 4 \times 15° = 60°$

$3x = 3 \times 15° = 45°$

10. Given that, In ΔABC,
$$2\angle A = 3\angle B = 6\angle C$$
$$2\angle A = 3\angle B$$
$$\Rightarrow \quad \frac{\angle A}{\angle B} = \frac{3}{2}$$
$$\Rightarrow \quad \angle A : \angle B = 3 : 2$$
Also, $3\angle B = 6\angle C$
$$\Rightarrow \quad \frac{\angle B}{\angle C} = \frac{6}{3}$$
$$\Rightarrow \quad \angle B : \angle C = 6 : 3$$
$$\angle A : \angle B = 3 : 2 = 9 : 6$$
$$\angle B : \angle C = 6 : 3$$
$$\therefore \quad \angle A : \angle B : \angle C = 9 : 6 : 3$$
Let $\angle A$, $\angle B$, $\angle C$ be $9x$, $6x$ and $3x$ respectively
$$\because \quad \angle A + \angle B + \angle C = 180°$$
$$\therefore \quad 9x + 6x + 3x = 180°$$
$$\Rightarrow \quad 18x = 180°$$
$$\Rightarrow \quad x = \frac{180°}{18°} = 10°$$
$$\therefore \quad \angle A = 9x = 9 \times 10° = 90°$$
$$\angle B = 6x = 6 \times 10° = 60°$$
$$\angle C = 3x = 3 \times 10° = 30°$$

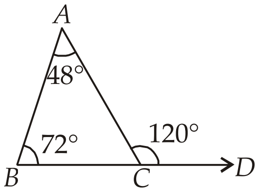
11. The given angles are $(x + 21)°$, $x°$ and $(2x - 45)°$.
$$(x + 21)° + x° + (2x - 45)° = 180° \quad (\because \text{ Angle sum property})$$
$$\Rightarrow \quad 4x° - 24° = 180°$$
$$\Rightarrow \quad 4x° = 180° + 24°$$
$$\Rightarrow \quad 4x° = 204°$$
$$\Rightarrow \quad x° = \frac{204°}{4} = 51$$
Hence, the value of x is 51.

EXERCISE 12.2

1. Let interior opposite angles of a triangle be $2x$ and $3x$. Therefore,

Exterior angle = sum of interior opposite angles of triangle

$\Rightarrow \quad 120° = 2x + 3x$



$\Rightarrow \quad 5x = 120°$

$\Rightarrow \quad x = \frac{120°}{5} = 24°$

Thus, the interior opposite angles are $2 \times 24° = 48°$ and $3 \times 24° = 72°$.

Now, in ΔABC,

$\angle A + \angle B + \angle C = 180°$

(Angle sum property)

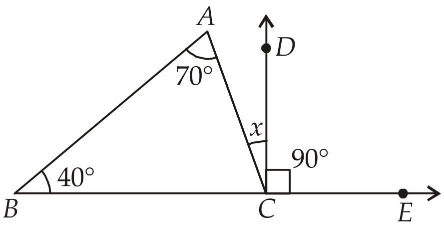
$\therefore \quad 48° + 72° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 120°$

$\Rightarrow \quad \angle C = 60°$

Hence, the measures of angles of the triangle are 48°, 72° and 60°.

2.



In ΔABC

$\angle ACE = \angle A + \angle B$

$= 70° + 40° = 110°$

Now, $\angle ACE = \angle ACD + \angle DCE$

($\because$ Adjacent angles)

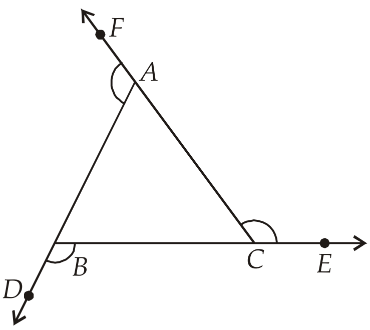
$110° = x + 90°$

$\Rightarrow \quad x = 110° - 90° = 20°$

Hence, the value of x is 20°.

3. Let ABC be a triangle. In ΔABC, AB, BC and CA are produced to D, E and F respectively to make exterior angles.

Now,



ext. $\angle A = \angle ABC + \angle ACB \quad \ldots(i)$

ext. $\angle B = \angle BAC + \angle ACB \quad \ldots(ii)$

ext. $\angle C = \angle BAC + \angle ABC \quad \ldots(iii)$

Adding equations (i), (ii) and (iii), we get,

ext. $\angle A$ + ext. $\angle B$ + ext. $\angle C$

$= 2\angle ABC + 2\angle ACB + 2\angle BAC$

$= 2(\angle ABC + \angle BAC + \angle ACB)$

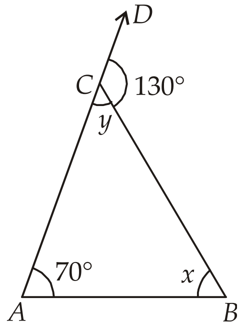
$= 2 \times 180°$

($\because$ Angle sum property)

ext. $\angle A$ + ext. $\angle B$ + ext. $\angle C = 360°$

Hence Proved.

4.



(i) In ΔABC,

$\angle BCD = \angle A + \angle B$

$\Rightarrow \quad \angle B = \angle BCD - \angle A$

$\Rightarrow \quad x = 130° - 70°$

$\Rightarrow \quad x = 60°$

$\therefore \quad \angle B = x = 60°$.

Now, $\angle A + \angle B + \angle C = 180°$

(Angle sum property)

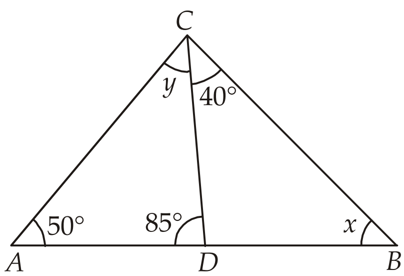
$70° + 60° + y = 180°$

$\Rightarrow \quad y = 180° - 130°$

$\Rightarrow \quad y = 50°$

Hence, $x = 60°$ and $y = 50°$.

(ii)



In ΔACD,

$\angle DAC + \angle ACD + \angle CDA = 180°$

($\because$ Angle sum property)

$\Rightarrow \quad 50° + y + 85° = 180°$

$\Rightarrow \quad y + 135° = 180°$

$\Rightarrow \quad y = 180° - 135°$

$\boxed{y = 45°}$

Now, in ΔBDC, $\quad \angle CDA = \angle BCD + \angle DBC$

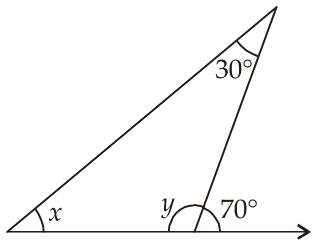
$85° = 40° + x$

$\Rightarrow \quad x = 85° - 40°$

$\Rightarrow \quad \boxed{x = 45°}$

Hence, $x = 45°$ and $y = 45°$.

5. In the given figure,



$\Rightarrow \quad 70° = x + 30°$

[Exterior angle property]

$\Rightarrow \quad x = 70° - 30°$

$\boxed{x = 40°}$

Now, $\quad y + 70° = 180° \quad$ (Linear pair)

$\Rightarrow \quad y = 180° - 70°$

$\Rightarrow \quad \boxed{y = 110°}$

Hence, $x = 40°$ and $y = 110°$.

6. Let the interior opposite angles of a triangle be $5x$ and $7x$.

Therefore,

Exterior angle = Sum of the interior opposite angles

$\therefore \quad 5x + 7x = 120°$

$\Rightarrow \quad 12x = 120°$

$\Rightarrow \quad x = 10°$

Then, $\quad 5x = 5 \times 10° = 50°$

$7x = 7 \times 10° = 70°$

Now,

third angle of the triangle $= 180° - (50° + 70°)$

$= 60°$

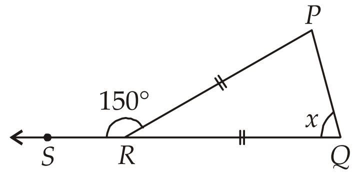
Hence, all the interior angles of the triangle are 50°, 70° and 60°.

7. In ΔPRQ, side QR is extended to S, and $RP = RQ$. Therefore,

$\angle P = \angle Q = x \quad$ (given)

Then, $\quad \angle PRS = \angle P + \angle Q$

$150° = x + x$



$\Rightarrow \quad 2x = 150°$

$\Rightarrow \quad x = \frac{150°}{2} = 75°$

Thus, $\quad \angle P = \angle Q = 75°$

Now, $\quad \angle P + \angle Q + \angle R = 180°$

($\because$ Angle sum property)

$\Rightarrow \quad 75° + 75° + \angle R = 180°$

$\Rightarrow \quad \angle R = 180° - 150°$

$\Rightarrow \quad \angle R = 30°$

Hence, $\angle P = \angle Q = 75°$, $\angle R = 30°$.

8. We know that, the sum of the two sides of a triangle is always greater than the third side.

i.e., the third side has to be less than the sum of the two sides.

Therefore,

the third side has to be $<$ (6 + 8) cm = 14 cm.

And, the third side is always greater than the difference of two sides. Then,

The third side has to be more than (8 – 6) cm = 2 cm.

Hence, the length of the third side could be any length greater than 2 cm but less than 14 cm.

9. (*i*) Sides are 6 cm, 3 cm and 2 cm.

Now, $6 + 3 > 2$, $3 + 2 < 6$, $6 + 2 > 3$

Since, $3 + 2 < 6$, so, it is not possible to draw a triangle with sides 6 cm, 3 cm and 2 cm.

(*ii*) Sides are 3 cm, 6 cm and 7 cm.

Now, $3 + 6 > 7$, $6 + 7 > 3$, $3 + 7 > 6$

Hence, it is possible to draw a triangle with sides 3 cm, 6 cm and 7 cm.

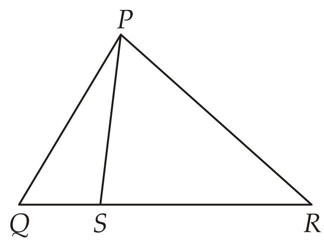
(*iii*) Sides are 10.2 cm, 5.8 cm and 4.5 cm.

Now, $10.2 + 5.8 > 4.5$, $5.8 + 4.5 > 10.2$,

$4.5 + 10.2 > 5.8$

Hence, it is possible to draw a triangle with sides 10.2 cm, 5.8 cm and 4.5 cm.

10.



In ΔPQS,

$$PQ + QS > PS \qquad \ldots(i)$$

Since, sum of the two sides of a triangle always greater than the third side.

Similarly, in ΔPSR

$$PR + RS > PS \qquad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

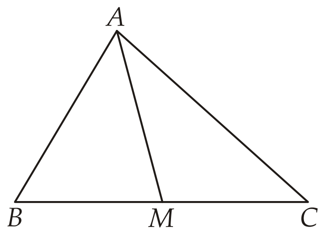
$$PQ + QS + PR + RS > PS + PS$$

$$\Rightarrow \quad PQ + (QS + RS) + PR > 2PS$$

$$\Rightarrow \quad PQ + QR + PR > 2PS$$

$$(\because \; QR = QS + RS)$$

11.



In ΔABM,

$$AB + BM > AM \qquad \ldots(i)$$

Since, sum of the two sides of a triangle is always greater than third side.

Similarly, in ΔAMC,

$$AC + CM > AM \qquad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

$$AB + BM + AC + CM > AM + AM$$

$$\Rightarrow \quad AB + 2BM + AC > 2AM$$

$$\because \; AM \text{ is the median, } \therefore \; BM = CM$$

$$\Rightarrow \quad AB + BC + AC > 2AM \qquad (\because \; BC = 2BM)$$

$$AB + BC + CA > 2AM$$

Hence proved.

12. Interior $\angle A = \frac{3}{7}$ of exterior $\angle A$

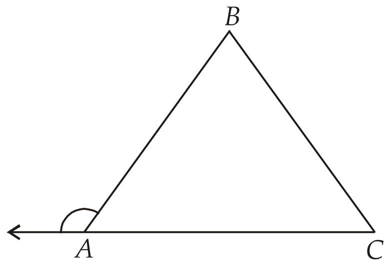
$\because$ Interior $\angle A$ + exterior $\angle A = 180°$ (Linear pair)

$$\Rightarrow \quad \frac{3}{7} \text{ exterior } \angle A + \text{exterior } \angle A = 180°$$

$$\Rightarrow \quad \frac{10}{7} \text{ exterior } \angle A = 180°$$

$$\Rightarrow \quad \text{exterior } \angle A = 180° \times \frac{7}{10} = 126°$$

$$\therefore \quad \text{exterior } \angle A = \frac{3}{7} \times 126° = 54°$$



Now, Exterior $\angle A = \angle B + \angle C$

$$\Rightarrow \quad 126° = \angle B + \frac{3}{4} \angle B$$

$$\left[\because \; 3\angle B = 4\angle C \;\; \Rightarrow \;\; \angle C = \frac{3}{4}\angle B\right]$$

$$\Rightarrow \quad \frac{7}{4}\angle B = 126°$$

$$\Rightarrow \quad \angle B = 126° \times \frac{4}{7} = 72°$$

Now, $\angle A + \angle B + \angle C = 180°$

$$\Rightarrow \quad 54° + 72° + \angle C = 180°$$

$$\Rightarrow \quad 126° + \angle C = 180°$$

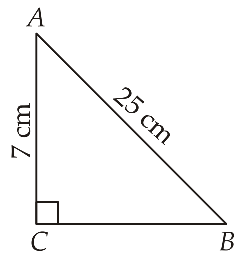
$$\Rightarrow \quad \angle C = 180° - 126° = 54°$$

Hence, $\angle A = 54°$, $\angle B = 72°$ and $\angle C = 54°$.

EXERCISE 12.3

1. In triangle ABC, $\angle C = 90°$.

By Pythagoras theorem,



$$AB^2 = AC^2 + BC^2$$

$$25^2 = 7^2 + BC^2$$

$$BC^2 = (625 - 49) \text{ cm}^2$$

$$BC^2 = 576 \text{ cm}^2$$

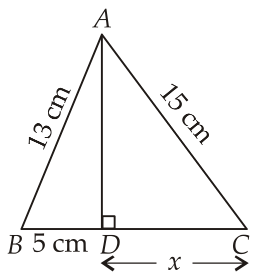
$$BC^2 = (24)^2 \text{ cm}^2$$

Hence, $BC = 24$ cm.

2. In right angled ΔABD, $\angle D = 90°$

$$AB^2 = BD^2 + AD^2$$

(By Pythagoras theorem)



$$\Rightarrow \quad AD^2 = AB^2 - BD^2$$

$$\Rightarrow \quad AD^2 = (13^2 - 5^2) \text{ cm}^2$$

$$\Rightarrow \quad AD^2 = (169 - 25) \text{ cm}^2$$

$$\Rightarrow \quad AD^2 = 144 \text{ cm}^2$$

$$\Rightarrow \quad AD^2 = (12)^2 \text{ cm}^2$$

$$\Rightarrow \quad AD = 12 \text{ cm}$$

Now, in right angled ΔADC, $\angle D = 90°$.

$$AC^2 = AD^2 + DC^2 \text{(By Pythagoras theorem)}$$

$$\Rightarrow \quad (15)^2 = (12)^2 + x^2$$

$$\Rightarrow \quad x^2 = (15^2 - 12^2) \text{ cm}^2$$

$$\Rightarrow \quad x^2 = (225 - 144) \text{ cm}^2$$

$$\Rightarrow \quad x^2 = 81 \text{ cm}^2$$

$$\Rightarrow \quad x^2 = (9)^2 \text{ cm}^2$$

$$\Rightarrow \quad x = 9 \text{ cm}$$

Hence, the value of x is 9 cm.

3. Let W be the window at the height of 8 cm from ground at point G, and L be the foot of ladder.

Then, $WG = 8$ m,

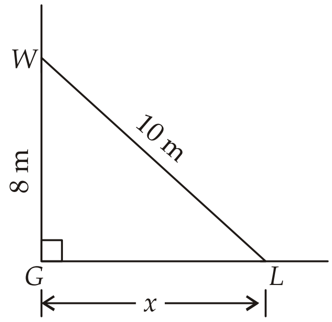
WL = 10 cm, $GL = x$ (given)

In right angled ΔWGL,

$$(WL)^2 = (WG)^2 + (GL)^2$$

(By Pythagoras theorem)

$$(10)^2 = (8)^2 + (GL)^2$$



$\Rightarrow \quad x^2 = (10)^2 - (8)^2$

$\Rightarrow \quad x^2 = 100 - 64$

$\Rightarrow \quad x^2 = 36$

$\Rightarrow \quad x^2 = 6^2$

$\Rightarrow \quad x = 6$

Hence, the value of x is 6 m.

4. (*i*) **3, 7, 9**

The largest side is 9. Then,

$$3^2 + 7^2 = 9 + 49 = 58$$

and $9^2 = 81$

Thus, $3^2 + 7^2 \neq 9^2$

Hence, 3, 7, 9 is not a Pythagorean triplet.

(*ii*) **7, 24, 25**

The largest side is 25. Then

$$(7)^2 + (24)^2 = 49 + 576 = 625$$

and $(25)^2 = 625$

Thus, $7^2 + (24)^2 = (25)^2$

Hence, 7, 24, 25 is a Pythagorean triplet.

(*iii*) **4, 5, 7**

The largest side is 7.

Then $4^2 + 5^2 = 16 + 25 = 41$

and $(7)^2 = 49$

Thus, $4^2 + 5^2 \neq 7^2$

Hence, 4, 5, 7 is not a Pythagorean triplet.

5. Let the third side be x cm. Then,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$(25)^2 = (15)^2 + x^2$$

(By Pythagoras theorem)

$\Rightarrow \quad x^2 = (25)^2 - (15)^2$

$\Rightarrow \quad x^2 = 625 - 225$

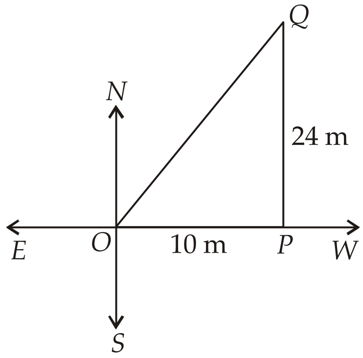
$\Rightarrow \quad x^2 = 400$

$\Rightarrow \quad x^2 = (20)^2$

$\Rightarrow \quad x = 20$

Hence, the length of the third side is 20 cm.

6.



In right angled ΔOPQ, $\angle P = 90°$

$\therefore \quad OQ^2 = OP^2 + PQ^2$

(By Pythagoras theorem)

$\Rightarrow \quad OQ^2 = (10\text{ m})^2 + (24\text{ m})^2$

$\Rightarrow \quad OQ^2 = (100 + 576)\text{ m}^2$

$\Rightarrow \quad OQ^2 = 676\text{ m}^2$

$\Rightarrow \quad OQ^2 = (26\text{ m})^2$

$\Rightarrow \quad OQ = 26$ m

Hence, the distance of man from the starting point is 26 m.

7. The sides of a triangle are 15 cm, 36 cm and 39 cm.

$\because$ The largest side is 39 cm.

Now,

$$(15)^2 + (36)^2 = (225 + 1296) = 1521$$

And $(39)^2 = 1521$

Thus, $(15)^2 + (36)^2 = (39)^2$

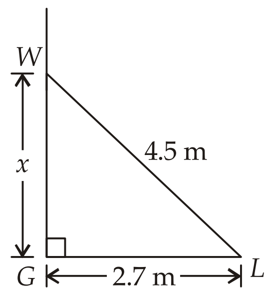
Hence, it is a right angled triangle.

8. Let WL be the ladder and L be the foot of the ladder. Let W be a point on the wall WG to which the ladder reaches.

Let the height of the wall WG be x m.

We have,

Length of ladder (WL) = 4.5 m, GL = 2.7 m

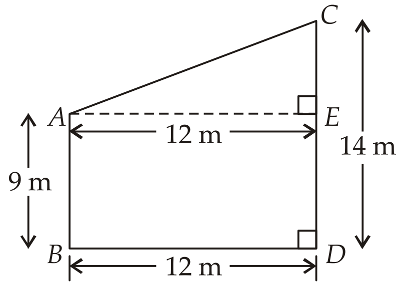


In right angled ΔWGL, by Pythagoras theorem.

$$(WL)^2 = (WG)^2 + (GL)^2$$
$$(4.5)^2 = x^2 + (2.7)^2$$
$$\Rightarrow \quad x^2 = 20.25 - 7.29$$
$$\Rightarrow \quad x^2 = 12.96$$
$$\Rightarrow \quad x^2 = (3.6)^2$$
$$\Rightarrow \quad x = 3.6$$

Hence, the height of the wall to which the ladder reaches is 3.6 m.

9. Let AB and CD be the two poles of heights 9 m and 14 m respectively and the distance between two poles BD = 12 m. Draw a line $AE \parallel BD$.



Then, $CE = DC - DE$
$= (14 - 9)$ m $\quad (\because\ DE = AB = 9$ m$)$
$= 5$ m

In right angled ΔAEC,

$$AC^2 = (AE)^2 + (CE)^2$$
$$\Rightarrow \quad AC^2 = (12)^2 + (5)^2$$
$$\Rightarrow \quad AC^2 = 144 + 25$$
$$\Rightarrow \quad AC^2 = 169$$
$$\Rightarrow \quad AC^2 = (13)^2$$
$$\Rightarrow \quad AC = 13$$

Hence, the distance between the tops of the two poles AB and CD is 13 m.

10. The sides of a triangle are of lengths 6.5 cm, 6 cm and 2.5 cm.

Now, the largest side is 6.5 cm. Then,

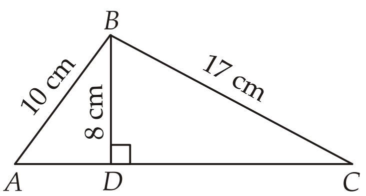
$$(6)^2 + (2.5)^2 = (36 + 6.25)$$
$$= 42.25$$

and $\quad (6.5)^2 = 42.25$

$$\therefore \quad (6)^2 + (2.5)^2 = (6.5)^2$$

Hence, it is a right angled triangle and the length of the hypotenuse is 6.5 cm.

11. In right angled ΔADB,



$$(AB)^2 = (AD)^2 + (BD)^2$$

(By Pythagoras theorem)

$$\Rightarrow \quad (AD)^2 = (10)^2 - (8)^2$$
$$\Rightarrow \quad (AD)^2 = (100 - 64)$$
$$\Rightarrow \quad (AD)^2 = 36$$
$$\Rightarrow \quad (AD)^2 = (6)^2$$

Thus, $\quad AD = 6$ cm

In right ΔBDC,

$$(BC)^2 = (BD)^2 + (DC)^2$$
$$(17)^2 = (8)^2 + (DC)^2$$
$$\Rightarrow \quad 289 = 64 + (DC)^2$$
$$\Rightarrow \quad DC^2 = 289 - 64$$
$$\Rightarrow \quad DC^2 = 225$$
$$\Rightarrow \quad DC^2 = (15)^2$$
$$\Rightarrow \quad DC = 15 \text{ cm}$$

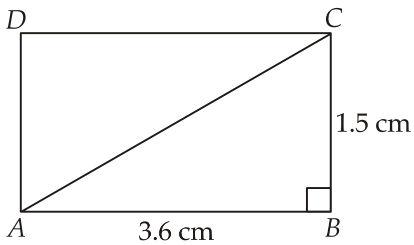
Hence, the length of $AC = AD + DC$

$$\Rightarrow \quad AC = 6 \text{ cm} + 15 \text{ cm}$$
$$\Rightarrow \quad AC = 21 \text{ cm}$$

12. Let $ABCD$ be a rectangle in which AB = 3.6 cm, BC = 1.5 cm.

In right angled ΔABC,

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(AC)^2 = (3.6 \text{ cm})^2 + (1.5 \text{ cm})^2$$
$$(AC)^2 = 12.96 \text{ cm}^2 + 2.25 \text{ cm}^2$$
$$(AC)^2 = (12.96 + 2.25) \text{ cm}^2$$



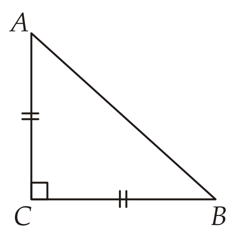
$$(AC)^2 = 15.21 \text{ cm}^2$$
$$(AC)^2 = (3.9 \text{ cm})^2$$
$$AC = 3.9 \text{ cm}$$

Hence, the length of the diagonal of the rectangle is 3.9 cm.

13. In isosceles triangle ABC, $\angle C = 90°$

$$AC = BC$$

AB cannot be equal to any of AC and BC as it is hypotanuse.



By Pythagoras theorem,

$$AB^2 = AC^2 + BC^2$$
$$= AC^2 + AC^2 \quad (\because\ BC = AC)$$
$$\boxed{AB^2 = 2AC^2}$$

Hence proved.

MULTIPLE CHOICE QUESTIONS

1. $\because$ $13^2 = 5^2 + 12^2$,

$39^2 = 36^2 + 15^2$

$26^2 = 24^2 + 10^2$

$\because$ $10^2 + 15^2 = 100 + 225 = 325$

$25^2 = 625$

$\therefore$ $10^2 + 15^2 \neq 25^2$

So, (10, 15, 25) is not a Pythagorean triplet.

Hence, option (*b*) is correct.

2. In ΔABC, $\angle ACD$ is an exterior angle.

$$\angle ACD = \angle BAC + \angle ABC$$
$$110° = 70° + x$$
$$\Rightarrow \quad x = 110° - 70° = 40°$$

Hence, option (*a*) is correct.

3. In ΔABC, $\angle B = 90°$

$AB = 5$ cm, $AC = 13$ cm, $BC = ?$

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$(13)^2 = (5)^2 + (BC)^2$$
$$\Rightarrow \quad BC^2 = 169 - 25 = 144$$
$$BC^2 = (12)^2$$
$$BC^2 = (12)^2$$

Thus, $BC = 12$ cm

Hence, option (*b*) is correct.

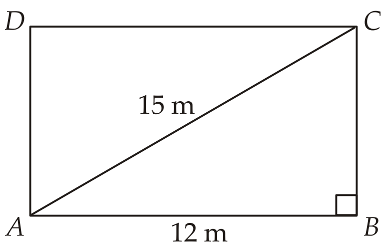
4. Let the width of the rectangle be x m.

Diagonal $AC = 15$ m, $AB = 12$ m

In ΔABC, $\angle B = 90°$.

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$



$$\Rightarrow \quad BC^2 = (15)^2 - (12)^2$$
$$\Rightarrow \quad x^2 = 225 - 144$$
$$\Rightarrow \quad x^2 = 81$$
$$\Rightarrow \quad x = 9$$

Hence, option (*d*) is correct.

5. Sum of the lengths of any two sides of a triangle is always greater than the third side.

$\because$ $5 + 6 = 11$

These cannot be the length of the sides of a triangle.

Hence, option (*b*) is correct.

6. The sum of exterior angles of a triangle is equal to 360°.

$\therefore$ $\angle 1 + \angle 2 + \angle 3 = 360°$

Hence, option (*b*) is correct.

7. Let the equal angles of a triangle be x. Then

$$x + x + 80° = 180° \quad (\because\ \text{Angle sum property})$$
$$2x + 80° = 180°$$
$$\Rightarrow \quad 2x = 100°$$
$$\Rightarrow \quad x = 50°$$

Hence, option (*c*) is correct.

8. By Pythagoras theorem,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$
$$50 = x^2 + x^2 \quad [\because\ \text{Base} = \text{Height} = x \text{ (let)}]$$
$$\Rightarrow \quad 2x^2 = 50$$
$$\Rightarrow \quad x^2 = 25$$
$$\Rightarrow \quad x = 5 \text{ cm}$$

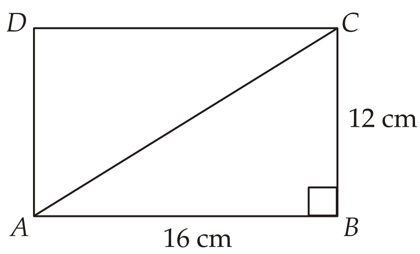
The length of each leg is 5 cm.

Hence, option (*b*) is correct.

9. Let $ABCD$ be the rectangle whose length $AB = 16$ cm and width $BC = 12$ cm, and AC be the diagonal.

In ΔABC, $\angle B = 90°$.

By Pythagoras theorem,



$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad AC^2 = (16)^2 + (12)^2$$
$$\Rightarrow \quad AC^2 = 256 + 144$$
$$\Rightarrow \quad AC^2 = 400$$
$$\Rightarrow \quad AC^2 = (20)^2$$
$$\Rightarrow \quad AC = 20$$

Thus, the length of each of its diagonals is 20 cm.

Hence, option (*a*) is correct.

10. $\because$ $CD \parallel BA$ and AC is transversal. Then

$$\angle BAC = \angle ACD \quad (\because\ \text{Alternate interior angles})$$

$\Rightarrow \quad \angle ACD = 50°$ $\quad (\because \angle BAC = 50°)$

Now, $\angle ACB + \angle ACD + \angle DCE = 180°$

$(\because BE$ is a straight line)

$\Rightarrow \quad \angle ACB + 50° + 60° = 180°$

$\Rightarrow \quad \angle ACB = 180° - 110° = 70°$

Hence, option (c) is correct.

11. ΔABC is a right angled triangle, right angle at B.

By Pythagoras theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

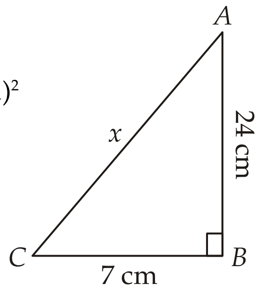
$x^2 = (24 \text{ cm})^2 + (7 \text{ cm})^2$

$\Rightarrow \quad x^2 = (576 + 49) \text{ cm}^2$

$\Rightarrow \quad x^2 = 625 \text{ cm}^2$

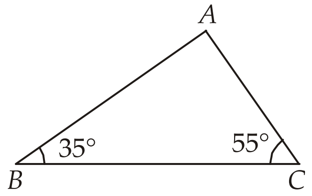
$\Rightarrow \quad x^2 = (25 \text{ cm})^2$

$\Rightarrow \quad x = 25 \text{ cm}$



Hence, option (d) is correct.

12.



In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + 35° + 55° = 180°$

$\Rightarrow \quad \angle A = 180° - 90°$

$\Rightarrow \quad \angle A = 90°$

Therefore,

ΔBAC is a right angled triangle.

Then, $(BC)^2 = (AB)^2 + (AC)^2$

Hence, option (b) is correct.

MENTAL MATHS CORNER

Mark the following statement as 'True' or 'False'.

1. A triangle with sides 2.5 cm, 2 cm and 1.5 cm is possible. **(True)**

$\because \quad 2.5 \text{ cm} + 2 \text{ cm} > 1.5 \text{ cm},$

$2 \text{ cm} + 1.5 \text{ cm} > 2.5 \text{ cm}$

and $\quad 2.5 \text{ cm} + 1.5 \text{ cm} > 2 \text{ cm}$

2. If AM is a median of ΔABC, then $AB + BC + CA > 2AM$. **(True)**

3. We can have a triangle with two right angles.

In this case, the sum of the angles of triangle will be more than 180°, which is not possible. **(False)**

4. Two angles of a triangle are 30° and 70°, then the third angle is 90°. **(False)**

$\because \quad 70° + 30° + 90° = 190° \neq 180°$

5. In an isosceles triangle, two angles are equal. **(True)**

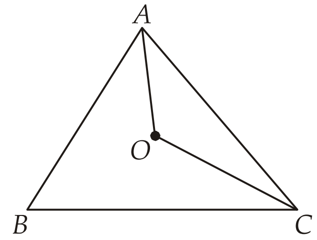
$\because$ If two sides of a triangle are equal, then the angles opposite to them are also equal.

6. The sum of the lengths of two sides of a triangle is always less than the third side. **(False)**

The sum of the lengths of two sides of a triangle is always greater than the third side.

7. O is any point in the interior of a ΔABC, then $OA + OC > AC$. **(True)**

$\because$ The sum of two sides of a triangle is always greater than its third sides.



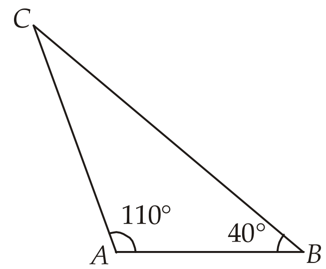
$\therefore$ In ΔOAC, $OA + OC > AC$

8. In a ΔABC, $\angle A = 110°$, $\angle B = 40°$, then the largest side is BC and the smallest side is AB.

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 110° - 40°$

$= 30°$



Side opposite to largest angle of a triangle is always greater than the other two.

BC is greatest and AB is smallest. **(True)**

9. A triangle can be drawn with sides 2.4 cm, 1.6 cm and 4 cm. **(False)**

$\because \quad 2.4 \text{ cm} + 1.6 \text{ cm} = 4 \text{ cm}$

So, it is not possible to draw a triangle with these sides.

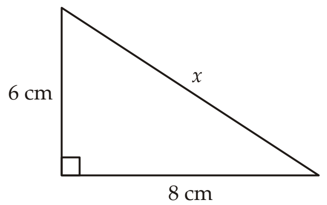
10. Three numbers a, b and c form a pythagorean triplet, if $a^2 + b^2 - c^2 = 0$. **(True)**

11. A triangle with two acute angles is not possible. **(False)**

12. Of all the line segments that can be drawn to a given line from a given point outside it, the perpendicular line segment is the shortest. **(True)**

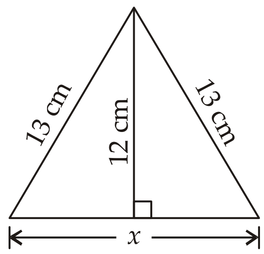
REVIEW EXERCISE

1. (*i*) In given right angled triangle by the Pythagoras theorem,



$$x^2 = (8\text{ cm})^2 + (6\text{ cm})^2$$
$$\Rightarrow \quad x^2 = 64\text{ cm}^2 + 36\text{ cm}^2$$
$$\Rightarrow \quad x^2 = (64 + 36)\text{ cm}^2$$
$$\Rightarrow \quad x^2 = 100\text{ cm}^2$$
$$\Rightarrow \quad x^2 = (10\text{ cm})^2$$
$$\Rightarrow \quad \boxed{x = 10\text{ cm}}$$

(*ii*) In the given triangle, two sides are equal. Then it is an isosceles triangle. Then, the altitude divides the triangle into two right angled triangle.



In any one of these right triangle,

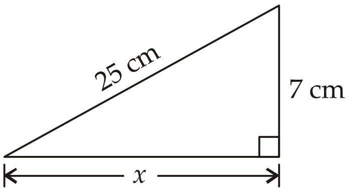
$$(13\text{ cm})^2 = (12\text{ cm})^2 + \left(\frac{x}{2}\right)^2$$

($\because$ By Pythagoras theorem)

$$\Rightarrow \quad \left(\frac{x}{2}\right)^2 = (169 - 144)\text{ cm}^2$$
$$\Rightarrow \quad \left(\frac{x}{2}\right)^2 = (25\text{ cm})^2$$
$$\Rightarrow \quad \frac{x}{2} = 5\text{ cm}$$

Therefore, $\boxed{x = 10\text{ cm}}$

(*iii*) In the given right angled triangle,



By Pythagoras theorem,

$$(25\text{ cm})^2 = (7\text{ cm})^2 + x^2$$
$$\Rightarrow \quad x^2 = (625 - 49)\text{ cm}^2$$
$$\Rightarrow \quad x^2 = 576\text{ cm}^2$$
$$\Rightarrow \quad x^2 = (24)^2\text{ cm}^2$$
$$\Rightarrow \quad \boxed{x = 24\text{ cm}}$$

2. Let the angles of a triangle be x, $2x$ and x. Therefore,

$$x + 2x + x = 180° \qquad \text{(Angle sum property)}$$
$$\Rightarrow \quad 4x = 180°$$
$$\Rightarrow \quad x = \frac{180°}{4} = 45°$$

Thus, the angles of a triangle are 45°, 90°, 45°. It is a right angled isosceles triangle.

(One of the angles is 90°)

It is an isosceles triangle. (Two angles are equal)

3. Let the third angle of a triangle be x.

Then, each of the two equal angles = $2x$

Therefore,

$$2x + x + 2x = 180° \qquad (\because \text{ Angle sum property})$$
$$\Rightarrow \quad 5x = 180°$$
$$\Rightarrow \quad x = \frac{180°}{5} = 36°$$

Hence, the angles of a triangle are 72°, 72° and 36°.

4. (*i*) 1 cm, 1 cm, 1 cm

Sum of the lengths of any two sides is always greater than the third side.

1 cm + 1 cm > 1 cm

Hence, it is possible to draw a triangle with the given sides.

(*ii*) 6 cm, 7 cm, 14 cm

$\because$ 6 cm + 7 cm < 14 cm

So, it is not possible to draw a triangle with the given sides.

(*iii*) 5 cm, 7 cm, 12 cm

$\because$ 5 cm + 7 cm = 12 cm

So, it is not possible to draw a triangle with the given sides.

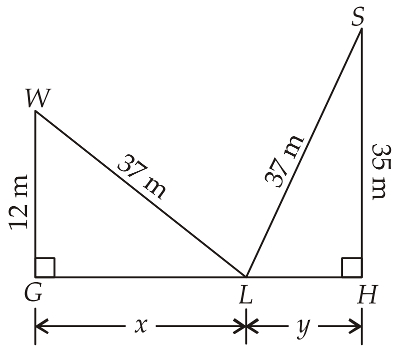
(*iv*) 2 cm, 10 cm, 15 cm

$\because$ 2 cm + 10 cm < 15 cm

So, it is not possible to draw a triangle with the given sides.

5. Let GH be the street and L be the foot of the ladder. Let S and W be the windows at the heights of 35 m and 12 m respectively from the ground. Then WL and SL are two positions of the ladder.

Let $GL = x$ m, and $LH = y$ m



Now, in ΔWGL, $\angle G = 90°$.

Then, by Pythagoras theorem,

$$(WL)^2 = (WG)^2 + (GL)^2$$
$$(37 \text{ m})^2 = (12 \text{ m})^2 + (x)^2$$
$$\Rightarrow \quad x^2 = (1369 - 144) \text{ m}^2$$
$$\Rightarrow \quad x^2 = 1225 \text{ m}^2$$
$$\Rightarrow \quad x^2 = (35 \text{ m})^2$$
$$\Rightarrow \quad \boxed{x = 35 \text{ m}}$$

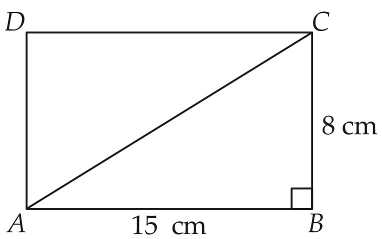
Again, in ΔSHL, $\angle H = 90°$.

$$(SL)^2 = (SH)^2 + (LH)^2$$
$$(37)^2 = (35)^2 + (y)^2$$
$$\Rightarrow \quad y^2 = (1369 - 1225) \text{ m}^2$$
$$\Rightarrow \quad y^2 = 144 \text{ m}^2$$
$$\Rightarrow \quad y^2 = (12 \text{ m})^2$$
$$\Rightarrow \quad \boxed{y = 12 \text{ m}}$$

Hence, the width of street $= x + y = (35 + 12)$ m
$= 47$ m.

6. Let $ABCD$ be a rectangle whose length is 15 cm and width is 8 cm. Let AC be its diagonal.

In ΔABC, $\angle B = 90°$.



By Pythagoras theorem,

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$\Rightarrow \quad (AC)^2 = (15 \text{ cm})^2 + (8 \text{ cm})^2$$
$$\Rightarrow \quad (AC)^2 = (225 + 64) \text{ cm}^2$$
$$\Rightarrow \quad (AC)^2 = 289 \text{ cm}^2$$
$$\Rightarrow \quad (AC)^2 = (17 \text{ cm})^2$$
$$AC = 17 \text{ cm}$$

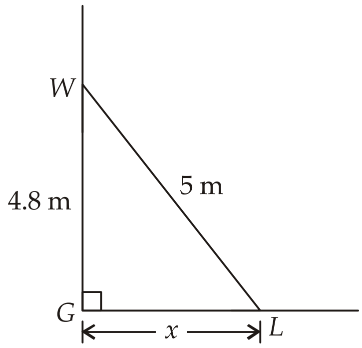
Hence, the length of the diagonal is 17 cm.

7. Let WL be a ladder reaches to W on the wall when set against it.

Let G be the foot of the wall.

Let the distance of the foot of ladder from the wall be x m. *i.e.*, $GL = x$ m

In right angled ΔWGL, $\angle G = 90°$.



By Pythagoras theorem,

$$(WL)^2 = (WG)^2 + (GL)^2$$
$$(5 \text{ m})^2 = (4.8 \text{ m})^2 + x^2$$
$$\Rightarrow \quad x^2 = (25 - 23.04) \text{ m}^2$$
$$\Rightarrow \quad x^2 = 1.96 \text{ m}^2$$
$$\Rightarrow \quad x^2 = (1.4 \text{ m})^2$$
$$\Rightarrow \quad x = 1.4 \text{ m}$$

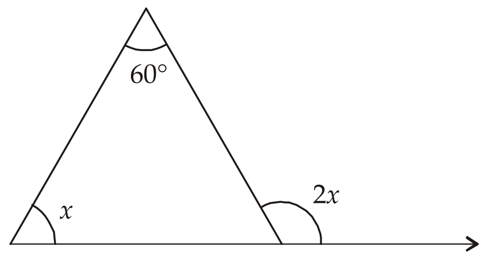
Hence, the distance of the foot of ladder from the wall is 1.4 m.

8. Let ABC be an isosceles right triangle. In which $\angle C = 90°$. Then the two equal angles are $\angle A$ and $\angle B$.

$$\angle A + \angle B + \angle C = 180°$$
$$\Rightarrow \quad x + x + 90° = 180° \quad [\because \angle A = \angle B = x \text{ (let)}]$$
$$\Rightarrow \quad 2x = 90°$$
$$\Rightarrow \quad x = 45°$$

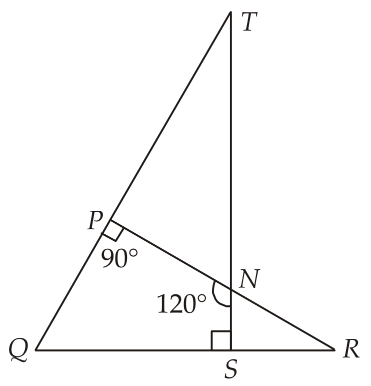
Hence, $\angle A = 45°$, $\angle B = 45°$, $\angle C = 90°$.

9. $\because$ Exterior angle = Sum of the two interior opposite angles.



$$\Rightarrow \quad 2x = x + 60°$$
$$\Rightarrow \quad 2x - x = 60°$$
$$\Rightarrow \quad x = 60°$$

10.



$\angle TNP + \angle PNS = 180°$ (Linear pair of angles)

$\angle TNP + 120° = 180°$

$\Rightarrow \quad \angle TNP = 60°$

In ΔTPN,

$\angle PTN + \angle TPN + \angle TNP = 180°$ (Angle sum property)

$\angle PTN + 90° + 60° = 180° \quad (\because \angle TPN = 90°)$

$\Rightarrow \quad \angle PTN = 180° - 150° = 30°$

(*i*) $\boxed{\angle QTS = \angle PTN = 30°}$

(*ii*) In ΔQTS,

$\angle QTS + \angle QST + \angle TQS = 180°$ (Angle sum property)

$30° + 90° + \angle TQS = 180°$

$\Rightarrow \quad \angle TQS = 180° - 120° = 60°$

Hence, $\boxed{\angle PQR = \angle TQS = 60°}$

(*iii*) In ΔPQR,

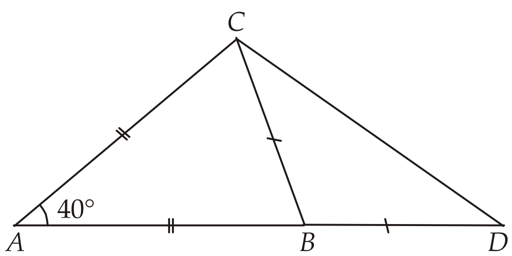
$\angle PQR + \angle QRP + \angle RPQ = 180°$ (Angle sum property)

$60° + \angle QRP + 90° = 180°$

$\Rightarrow \quad \angle QRP = 180° - 150°$

$\Rightarrow \quad \boxed{\angle QRP = 30°}$

11. (*i*) In ΔABC, $AC = AB$,



$\therefore \quad \angle ABC = \angle ACB = x$ (let)

Then,

$\angle BAC + \angle ACB + \angle ABC = 180°$

$\Rightarrow \quad 40° + x + x = 180°$

$\Rightarrow \quad 2x = 180° - 40° = 140°$

$\Rightarrow \quad x = \dfrac{140°}{2} = 70°$

Hence, $\boxed{\angle ACB = 70°}$

(*ii*) In ΔBDC, $BC = BD$

$\therefore \quad \angle BCD = \angle CDB = y$ (let)

$\angle BCD + \angle CDB + \angle DBC = 180°$

$y + y + 110° = 180°$

$\because \quad \angle DBC = \angle ACB + \angle CAB$

$= 70° + 40° = 110°$

$\Rightarrow \quad 2y = 180° - 110°$

$\Rightarrow \quad 2y = 70°$

$\Rightarrow \quad y = 35°$

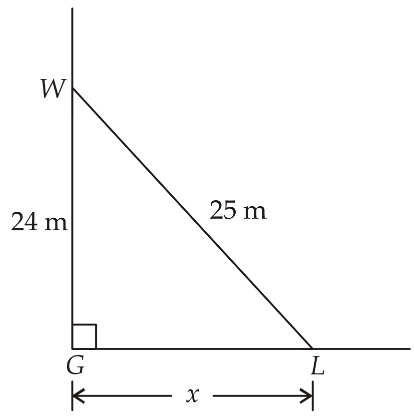
Hence, $\boxed{\angle CDB = 35°}$

12. Let WL be a ladder reaches to W on the wall when set against it.

Let G be the foot of the wall.

Let the distance of the foot of ladder from the wall be x m. *i.e.*, $GL = x$ m

In right angled ΔWGL, $\angle G = 90°$.



In ΔWGL, $\angle G = 90°$

By Pythagoras theorem,

$(WL)^2 = (WG)^2 + (GL)^2$

$(25 \text{ m})^2 = (24 \text{ m})^2 + x^2$

$\Rightarrow \quad x^2 = (625 - 576) \text{ m}^2$

$\Rightarrow \quad x^2 = 49 \text{ m}^2$

$\Rightarrow \quad x^2 = (7)^2 \text{ m}^2$

$\Rightarrow \quad x = 7 \text{ m}$

Hence, the distance between lower end of the ladder and base of the wall is 7 m.

HOTS QUESTIONS

1. Let one of the interior opposite angles be x. Then

Exterior angle $= 2x$

Now, Exterior angle $= x + 60°$

$\Rightarrow \quad 2x = x + 60°$

$\Rightarrow \quad 2x - x = 60°$

$\Rightarrow \quad x = 60°$

$\because$ Both the interior angles are equal.

Third angle $= 180° - (60° + 60°)$

$= 180° - 120° = 60°$

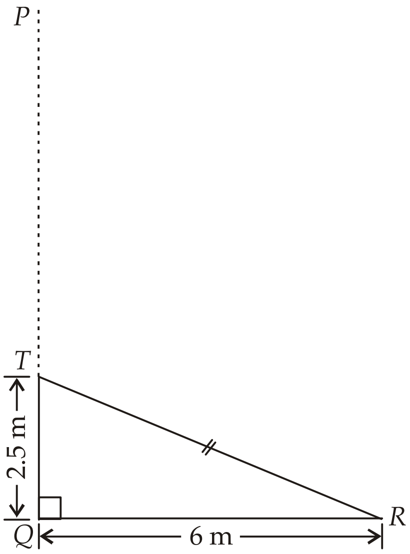
Thus, all the angles of the triangle are equal *i.e.* 60°.

Hence, it is an "equilateral triangle".

2. Let PQ be a tree of height $(2.5 + x)$ m before it is broke at point T.

i.e. $\quad PT = TR = x$ m

Let the top P touch the ground at R after it broke.



In ΔTQR, $\angle Q = 90°$

By Pythagoras theorem,

$(TR)^2 = (TQ)^2 + (QR)^2$

$\Rightarrow \quad x^2 = (2.5 \text{ m})^2 + (6 \text{ m})^2$

$\Rightarrow \quad x^2 = (6.25 + 36) \text{ m}^2$

$\Rightarrow \quad x^2 = 42.25 \text{ m}^2$

$\Rightarrow \quad x = 6.5$ m

Thus, $\quad PT = TR = 6.5$ m

Hence, the height of the tree $= PT + TQ$

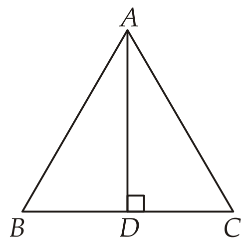
$= (6.5 + 2.5)$ m

$= 9$ m

3. (*i*) $\because$ ABC is an equilateral triangle. Then

$AB = BC = CA$

i.e., $\angle A = \angle B = \angle C = 60°$



Now, $AD \perp BC$.

In ΔABD,

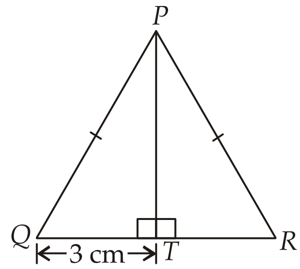
$\angle ABD + \angle BAD + \angle ADB = 180°$

$\Rightarrow \quad 60° + \angle BAD + 90° = 180°$

$\Rightarrow \quad \angle BAD = 180° - 150°$

$\angle BAD = 30°$

(*ii*)



Let the equal angles of isosceles triangle be x.

$\angle Q = \angle R = x$

Now,

$\angle P + \angle Q + \angle R = 180°$

$\Rightarrow \quad \angle P + x + x = 180°$

$\Rightarrow \quad \angle P = 180° - 2x$

Now, an altitude forms a right angle with the base it intersects.

Thus, there are 2 right angles formed at the foot of the altitude PT.

In ΔPTQ,

$\angle QPT + \angle PTQ + \angle TQP = 180°$

$\angle QPT + 90° + x = 180°$

$$\angle QPT = 90° - x = \frac{1}{2}(180° - 2x)$$

Similarly, $\quad \angle RPT = 90° - x$

These angles are half the size of $\angle QPR$

So, $\angle QPR$ is bisected by the altitude.

So, it bisects the side QR also.

Hence, $QT = RT$

$\therefore \quad RT = 3$ cm.